Unit - 4

Introduction to vehicle Dynamics

- Vehicle Dynamics encompasses – Ships, aeroplane, railroad, truck etc.

- Performance of vehicle – accelerating, braking, cornering and ride
  - Forces acting vehicle
  - Study their performance

- Handling – turning, cornering or directional response

- vehicle dynamics can be accomplished at two levels
  - empirical (trial & error) :- It may lead to failure
  - analytical
    - useful method
    - This model can be represented by set of equtns that relate forces or motions of interest to control inputs & vehicle performance.

Today with the computational power available in desktop and mainframe Computer a major shortcomes of analytical approach can be ~~overco~~ overcome.

Now we can make model, Simulation & evaluations of the behaviour of model.

Fundamental approach to modeling.

Vehicle Dynamics → movement of vehicles.
like,
- braking
- acceleration
- ride
- turning.

Dynamics behaviour can be studied by forces imposed on vehicle. from tires, gravity and aerodynamics. at different conditions like particular maneuver and trim condition.

Lumped mass

vehicles all components can be represented by one lumped mass located at its C.G. with appropriate mass & Inertia properties

For, acceleration, braking, turning analysis ⇒ lumped mass (vehicle)

For, ride analysis.. — ~~wheels~~ lumped mass
- wheels (Unsprung mass)
- vehicle (Sprung mass)

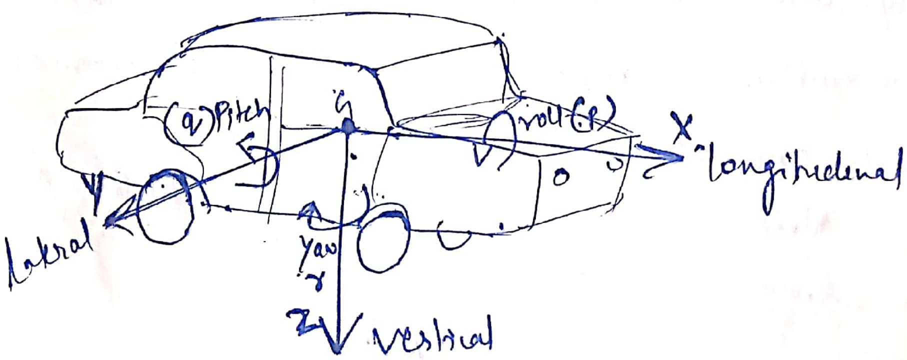


Fig:- vehicle axis system (By SAE Convention)

* vehicle fixed Co-ordinate system

The vehicle motions are defined with reference to a right orthogonal coordinates system called vehicle fixed co-ordinate system, which originate at C.G.

It is by SAE Conventions.

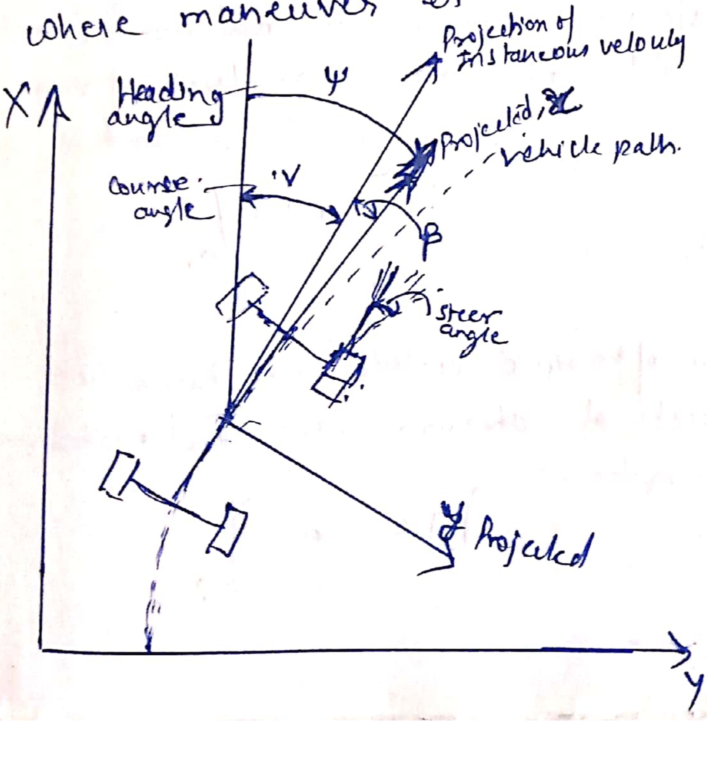
- x → Forward / Longitudinal
- y → lateral out right side of vehicle
- z → Downward w.r.t vehicle
- p → roll velocity ~~to~~ about x-axis
- q → Pitch — " — " — y-axis
- r → yaw — " — " — z-axis

} motion variables

* Earth fixed Corordinate System

vehicle attitude and trajectory through the course of maneuver are defined w.r.t right hand orthogonal axis system fixed on ~~earth~~ earth.

It is normally selected to ~~co-ordinate~~ coincide with the vehicle fixed coordinate system at the point where maneuver is started.



- X → Forward travel
- Y → Travel to the right
- Z → vertical travel (Positive downward)
- ψ → Heading angle (Angle between x & X)
- ν → Course Angle (Angle between the vehicle velocity & X-axis)
- β → Side slip angle (Angle vehicle velocity vector & x-axis)

Euler's Angle → Angle Relationship of vehicle fixed co-ordinate system to the earth fixed co-ordinate system is defined by Euler's Angle.

Euler's angle are determined by a sequence of three angular rotations

- yaw (around z-axis)
- Pitch (around y-axis)
- roll (around x-axis)

Earth fixed system.

Thus three angle obtained are Euler's angle.

Forces

- Positive Force in longitudinal direction (x-axis)
- Force corresponding to load on tires, act in upward direction (therefore negative z-direction)

Newton's 2nd law

(A) Translational System :- Sum of external forces acting on body is product of mass & acceleration in that direction

i.e $\boxed{F = m.a}$

(B) Rotational System :- sum of torque acting on body is Product of rotational M.I and rotational acceleration

$\boxed{T = I\alpha}$

* vehicle Springing System

Suspension system consist

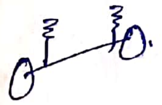
- Softness in riding
- handling ability.
- mini. wear to tyres & other parts

Functions

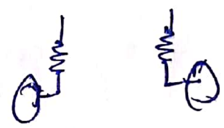
1. absorb large or small loads. & ~~ed~~ shocks
2. To minimize Pitch & roll.
3. reduce unspring mass
4. reduce Impact stress

Suspension system

- Conventional System
  {wheels mounted axle Suspended by spring}



- Independent System.
  { ~~suspended~~ Spring suspended independently on wheels }



* vehicle dynamics & Suspension Requirement

dynamic forces – Braking forces, Accelerating forces & Steering forces.

This forces depend upon frictional Contact between road & tyres

If Dynamic load exceed frictional force the Contact is lost. & hence Tyre may skid or slip.

- Ideally the tyre should contact squarely with ground, but it is not possible due to irregularity on road, wind, required directional control, changes in weight, acceleration and braking.

  - large bumps are absorbed by suspension system
  - Small — " —"—" —" — tyre itself.

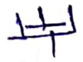
- The suspension system must be such that it must absorb shocks and also keep the tyre in contact with road.

- when spring in suspension system compresses it is called as "Jounce", when the spring is released to neutral position it is called "rebound".

Generally the Jounce & rebound rate is 60 to 70 cycles per min.

Any material can be looked form three simple models

① spring 

② dashpot 

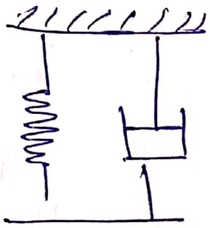
③ friction 

suppose we say any mtrl is purely elastic than it can be depicted by spring.

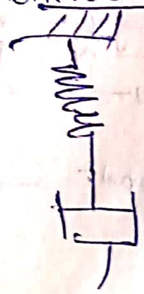
- Elastomers are having elastic & viscous behaviour
- Elastomers can be depicted as mode of spring & dashpot.

this can be represented by

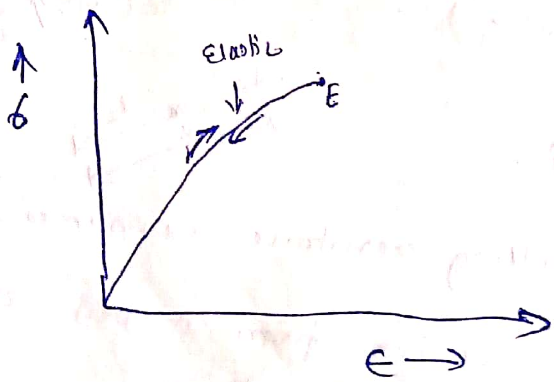
Kelvin model

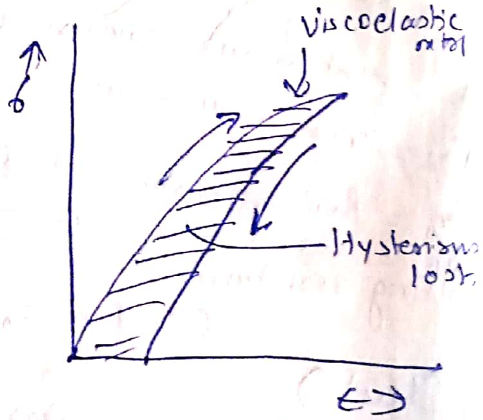


maxwell model



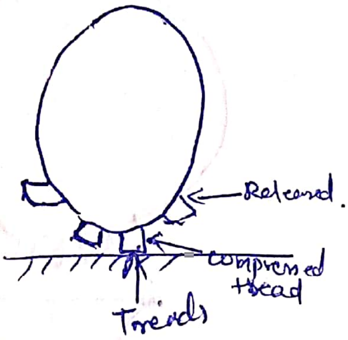
- For e.g. metal taken to plastic region than the metal can be modeled by spring & friction element
- Elastic mtrl is not always linear, it may be non-linear mtrl.





The amount of energy is lost.
At end of loading there is residual strain after loading but it will

- Come back after particular time
- So time & frequency are important
- mtrl. behaviour is affected by rate & frequency of loading.

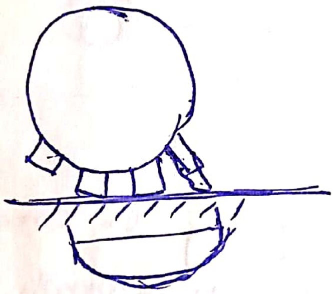


Tyre treads gets compressed & released hence ~~the~~ hence there is loading & unloading cycle go on

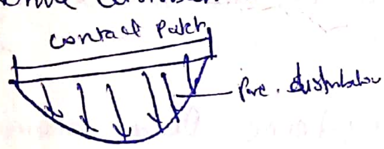
The Hysteresis loss is compensated by engine

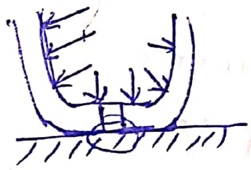
Contact Patch of tyre :-

- Pressure distribution at the contact.
  - Tyres we uses are pneumatic tyres
  - Tyres are inflate to a particular pressure.



Stationary tyre.
Pressure distribution

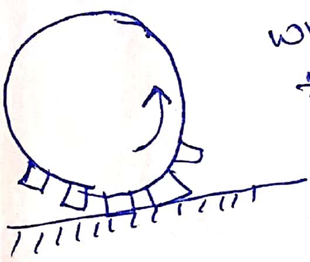




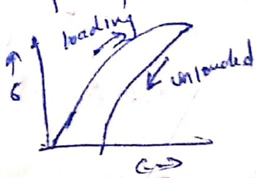
let us take small infinite element on tyre.

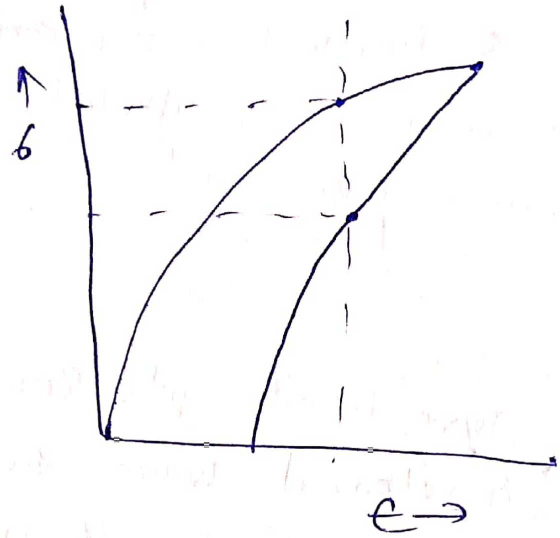
Contact pressure = Inflatation Pressure.

but practically, contact pressure is never uniform due to bending of side walls.

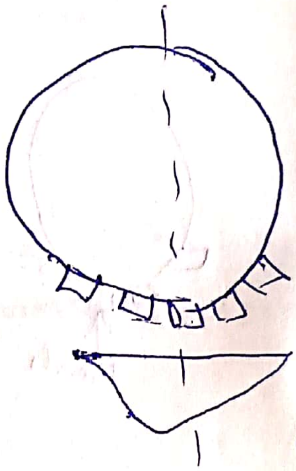


When rolling tyre is there.
The block/tread gets loaded & unloaded. ~~this~~ There is hysteresis developed.
Similarly the size ~~is all~~ of tyre is also loaded & unloaded.

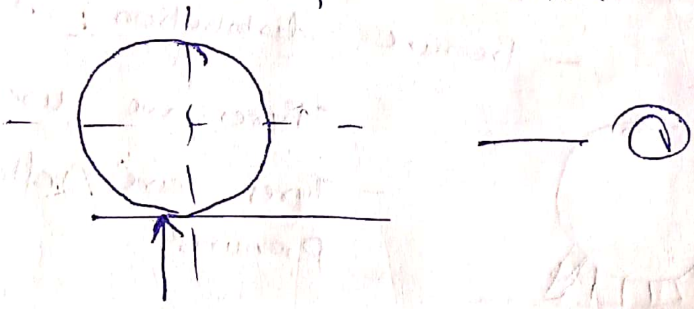




For same strain, the loading stress & unloading stress is different so,

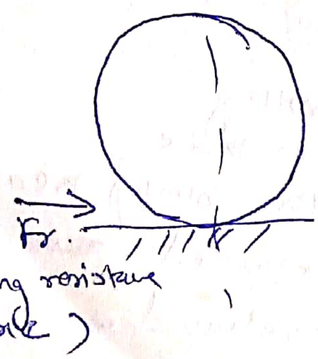


Due to this change in pressure distribution the load acts away from centre



Hence this force is now going to create a torque or moment which is opposite to motion & this force is known as rolling resistance force.

we are again replace this moment



(Rolling resistance force)

So we can say, that the rolling resistance force will be equal to weight W (from fig (1).)

$$\boxed{F_r = f_r \cdot W}$$

F_r = rolling resistance force
f_r = — u — coeff.
W = wt. of vehicle

4

* Performance characteristics of Road vehicles

~~Force of~~

(I) Drawbar - Pull and Drawbar Power :-

- The off road vehicles designed for traction (i.e tractors) the drawbar performance is of prime importance.
- It represents ability of vehicle to push or pull.
- like construction & earth moving equipment, agriculture

- $F_d \leftarrow$ drawbar pull.

$$\boxed{F_d = F - \Sigma R}$$

$F =$ Tractive force/effort. developed by running gear.

$\Sigma R =$ Resultant resisting force acting on vehicle.

Resisting force may include

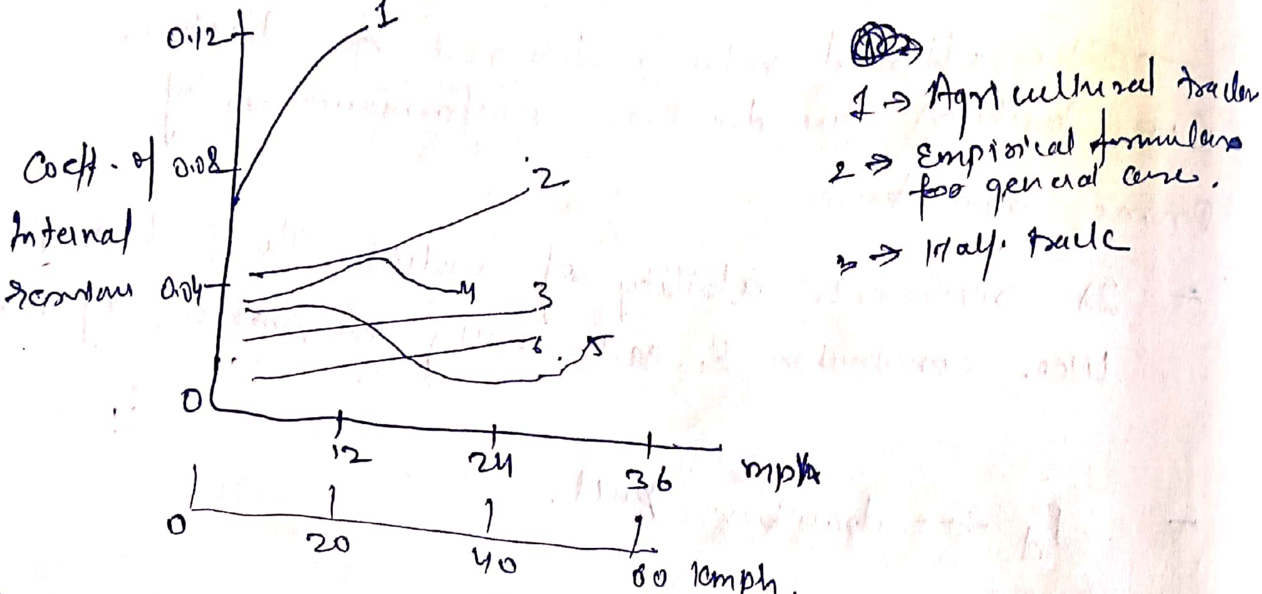
1. Internal resistance of Running gears
2. Resistance due to vehicle terrain. interaction
3. Obstacle resistance
4. grade resistance
5. Aerodynamic resistance.

(1) Internal Resistance of Running gear :-

It is due to hysterisis loss in tyres

(2) Resistance due to vehicle-Terrain Interaction

The resistance due to compacting the terrain and bulldozing effect.



(3) Ground Obstacle Resistance :-

The obstacle resistance may be considered as a resisting force, usually variable in magnitude acting parallel to the ground at certain effective height.

(4) Aerodynamic Resistance

not significant for speed below 48 km/h

	C_D	Area (frontal)
(A) For heavy vehicles (like tanks)	1.0	6–8 m^2
(B) For tanks (wt = 50 tonnes)	1.17	6.5 m^2

(II) Vehicle Powerplant and Transmission Characteristics

(A) Powerplant Characteristics

– For ideal performance characteristics of a Power plant are constant power output over full range speed.

- Engine output ~~Speed~~ torque varies with speed hyperbolically (S)

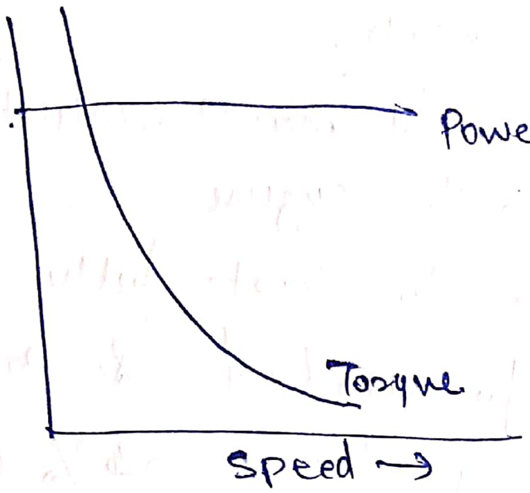


Fig:- Ideal Performance characteristics of vehicular power plant

- This provide vehicle with high tractive effort at low speed. (where demand for acceleration, drawbar pull, grade climbing is high)

- I.C engine despite less favourable performance characteristics as compared to electric motor it is used because of relative high power to weight ratio, good fuel economy, low cost, easiness to start.

(2) Transmission characteristics

- As we know that power-torque-speed characteristics are not suitable for direct vehicle propulsion. hence transmission is required.

Types
- manual gear Transmission
- Automatic Transmission

(A) Manual gear transmission

Requirements :-

(1) To Acheive desired max^m vehicle speed with an appropriate engine

(2) To be able to start fully loaded, in both forwarded & reverse direction on steep gradient 33% (1 in 3).

To be able to maintain speed of 88-96 km/h on a gentle slope such as 3%.

(3) To properly match the characteristics of engine to achieve desired fuel economy & acceleration characteristics

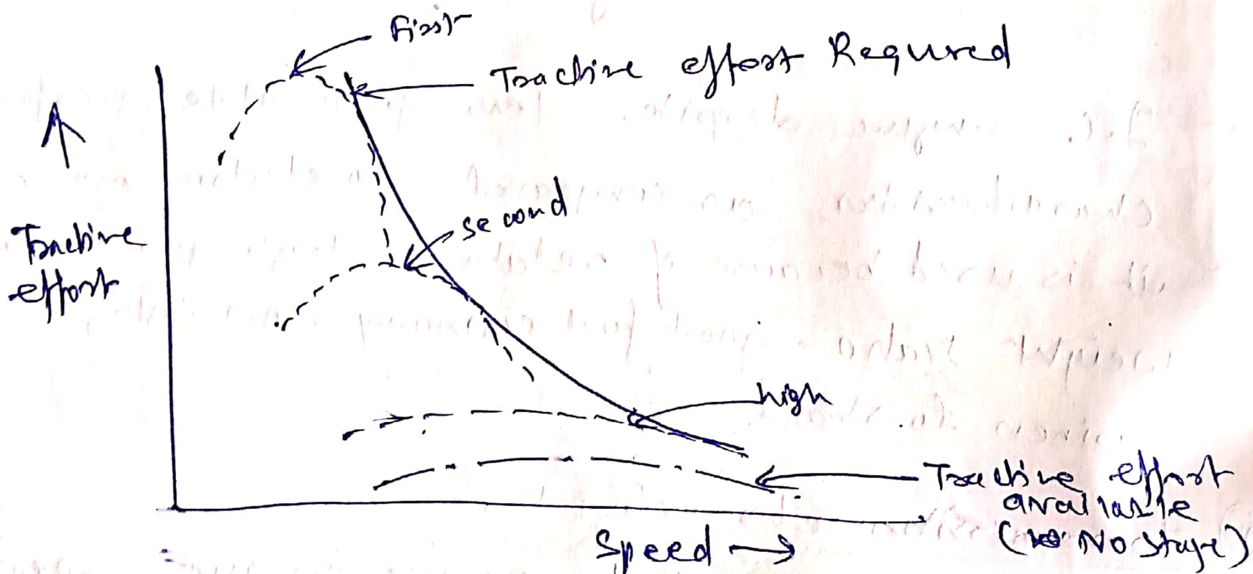


Fig :- Tractive effort vs speed characteristics of a passenger car.

(B) Automatic Transmission :-

An automatic transmission is usually used with a torque Converter.

Elements of torque Converter
- Pump (impeller)
- Turbine
- Reactor.

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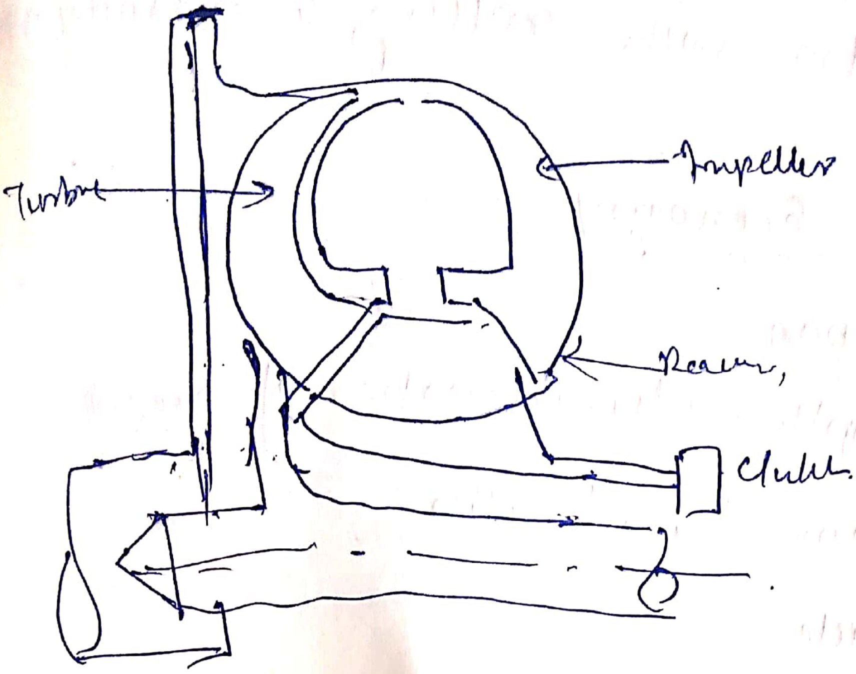


Advantages

① When properly matched it will not stall engine

② provide flexible coupling between engine & wheels.

③ Together with multispeed gearbox it provides torque - speed characteristics that approach the ideal.

Ⅲ Gradability

— Gradability is defined as the maximum ~~angle~~ grade a vehicle can negotiate at a given steady speed.

→ It is intended for evaluation of performance of heavy commercial vehicle & offroad vehicles.

- On a slope at a constant speed Traction force has to overcome grade resistance also with rolling & aerodynamic resistance.

(IV) operating fuel Economy

It depend upon

- Fuel consumption characteristics of Engine
- Transmission characteristics
- wt. of vehicle
- aerodynamic resistance
- rolling resistance of tires
- driving cycle (conditions)
- driver behaviour.

Aerodynamic forces & moments

- fuel economy ↑ then reduce (1) aerodynamic resistance
  (2) Inertia resistance
  (3) Rolling resistance.

- For vehicle at 80 km/h,
  aerodynamic resistance is more than rolling resistance & transmission resistance.
  So at high speed, high aerodynamic resistance

- Sources of aerodynamic resistance
  - exterior of vehicle body (90%)
  - Flow through radiator (10%)

- exterior Aerodynamic ~~pres~~ on vehicle body
  - normal pressure (90%) (pressure drag)
  - Shear stress on surface of vehicle (10%) (skin resistance)

- Aerodynamic resistance

$$\boxed{D_A = \frac{\rho}{2} C_D A_f V_r^2}$$

ρ = density of air

C_D = Coeff. of aerodynamic resistance.

A_f = characteristic Area of vehicle in direction of travel.

V_r = rel. speed of wind w.r.t vehicle.

- This ~~Aerodynamic~~ power required to overcome aerodynamic resistance ~~times~~ increases with cube of speed.

The frontal area and vehicle mass may be approximately taken as 18

$$\boxed{A_f = 1.6 + 0.00056\,(m_v - 765)}$$

where

A_f = frontal area in m^2

m_v = mass of vehicle in kg.

- C_D can be practically obtained by wind tunnel i.e by proper scaling the vehicle in a model.

In addition to the shape of vehicle body, attitude of the vehicle defined by the angle of attack (i.e angle between longitudinal axis of vehicle & the horizontal), ground clearance, loading conditions and operational factors — (such as radiator open or blanked window open or close).

Aerodynamic pitching moment

It also affect the behaviour of a vehicle. This moment is the resultant of the moments of aerodynamic resistance and aerodynamic lift about the C.G of vehicle

Due to this significant transfer of load may occur from one axle to other.

Thus it affect performance & directional control & stability

$$\boxed{M_a = \frac{\rho}{2} C_m A_f L_c V_r^2}$$

where

C_m = coeff. of aerodynamic pitching (obtained from wind tunnel)

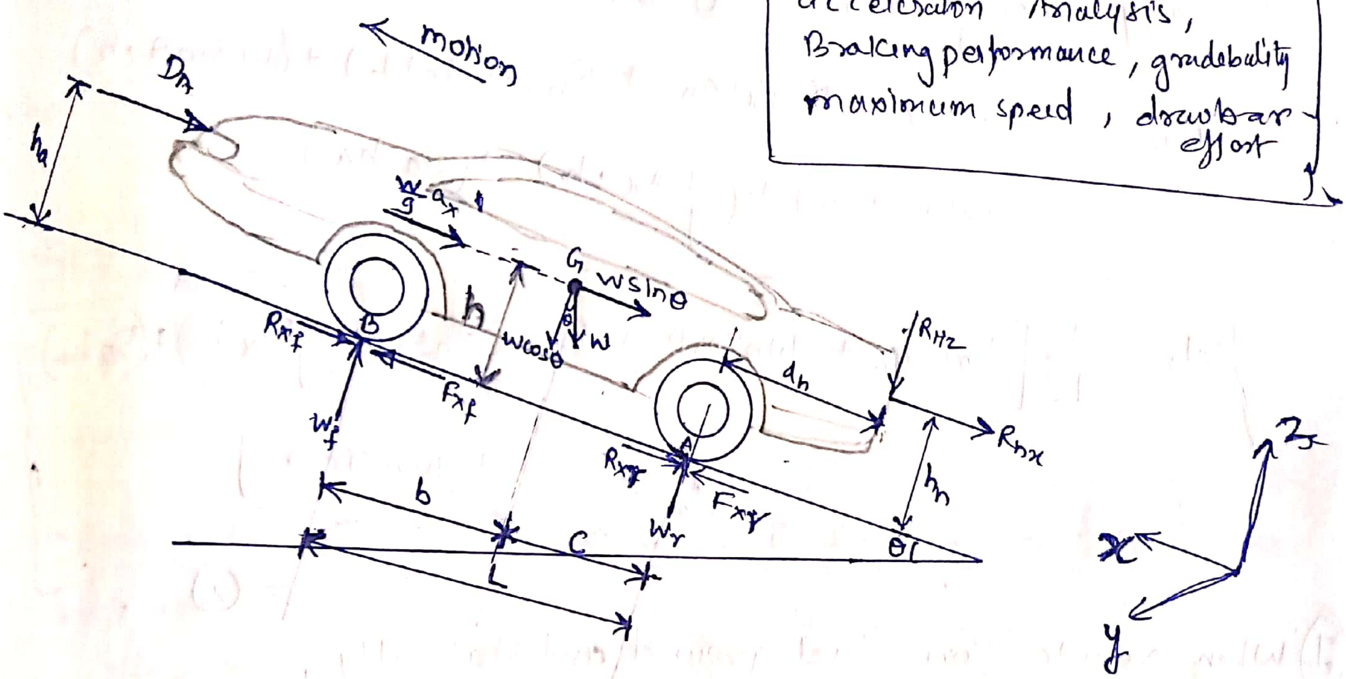
L_c = characteristic length (wheel base length)

For Cars, $C_m = 0.05$ to 0.20

* Dynamic Axle loads

12

By applying Newton 2nd law → Axle Forces determined → It determines Tractive efforts on axle → acceleration Analysis, Braking performance, gradeability, maximum speed, drawbar effort



$W \rightarrow$ wt. of vehicle @ C.G.

$\frac{W}{g}a_x \rightarrow$ Inertia Force opposite to direction of acceleration

$W_f, W_r \rightarrow$ Force on tire normal to road. (dynamic wt. carried on front and rear wheels)

$F_{xf}, F_{xr} \rightarrow$ Tractive Force (acting parallel to ground plane)

$R_{xf}, R_{xr} \rightarrow$ rolling resistance Force (—— " —— " —— " ——)

$D_A \rightarrow$ aerodynamic Force on vehicle (acting @ a point h_A above ground)

$R_{hz}, R_{hx} \rightarrow$ vertical and longitudinal Forces acting at the hitch when towing a trailer (or Drawbar Pull).

Taking moment @ A, Forces on B can be calculated.

$$\Sigma M_A = 0 = (W_f \cdot L) + (D_A \cdot h_a) + \left(\frac{W}{g}a_x\right)h + (R_{hx} \cdot h_h) + (R_{hz} \cdot d_h) + (W\sin\theta \cdot h) - (W c \cos\theta) = 0$$

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For uphill, θ is positive so, $\sin\theta$ is positive
For downhill, θ is negative so, $\sin\theta$ is negative

So,

$$\boxed{W_f = \frac{1}{L}\left[Wc\cos\theta - R_{hx}h_h - R_{hz}d_h - \frac{W}{g}a_x h - D_A h_a - Wh\sin\theta\right]} \quad \text{--- (1)}$$

Similarly taking moment @ B.

$$\Sigma M_B = 0 = -W_r L + R_{hx}\cdot h_h + R_{hz}\cdot(d_h+L) + (W\sin\theta\cdot h)$$
$$+ (W\cos\theta\cdot b) + \left(\frac{W}{g}a_x\cdot h\right) + (D_A\cdot h_a)$$

$$\boxed{W_r = \frac{1}{L}\left[R_{hx}\cdot h_h + Wb\cos\theta + R_{hz}(d_h+L) + \left(\frac{W}{g}a_x\cdot h\right) + (D_A h_a) + Wh\sin\theta\right]} \quad \text{--- (2)}$$

(I) When vehicle on level ground and statically,

$\theta = 0$, $\sin 0^\circ = 0$ and variables R_{hx}, R_{hz}, a_x and D_A are 0.

So,

$$\boxed{W_{fs} = \frac{1}{L}[Wc]} \quad \text{--- (3)}$$

$$\boxed{W_{rs} = \frac{1}{L}(Wb)} \quad \text{--- (4)}$$

(II) When vehicle on low-speed Acceleration & level ground

$D_A = 0$; $R_{hx} = 0$, $R_{hz} = 0$ (presuming no trailer)

$$W_f = W\left(\frac{c}{L} - \frac{a_x h}{gL}\right)$$
$$= \frac{Wc}{L} - \frac{W a_x h}{gL}$$

from equtn ③.

$$\boxed{W_f = W_{fs} - \frac{W a_x h}{gL}}$$

Similarly

$$W_r = W\left(\frac{b}{L} + \frac{a_x h}{gL}\right)$$

$$W_r = \frac{Wb}{L} + \frac{W a_x h}{gL}$$

from equtn ④,

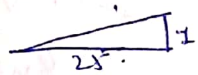
$$\boxed{W_r = W_{rs} + \frac{W a_x h}{gL}}$$

Thus when vehicle accelerate, load is transfered from the front axle to the rear axle in proportion to acceleration and the ratio of C.G height to wheel base.

(III) Influence of grade on axle.

Grade is defined as "rise" over the "run". This ratio is the tangent of grade angle θ

For highway grade is $= 4\%$ (1 in 25)



For primary & seconday rodes $= 10$ to 12%

$\frac{rise}{run} \times 100 = \%$ grad

$\cos\theta \approx 1$
$\sin\theta \approx \theta$ } for small angles.

Thus a positive grade causes load to transfer from front to rear axle.

Thus, — taking $a_x = 0$, $R_{hx} = 0$, $R_{hz} = 0$. $D_a = 0$,

$$W_f = W\left(\frac{c}{L} - \frac{h}{L}\theta\right)$$

$$\boxed{W_f = W_{fs} - \frac{Wh\theta}{L}}$$

$$W_r = W\left(\frac{b}{L} + \frac{h}{L}\theta\right)$$

$$\boxed{W_r = W_{rs} + \frac{Wh\theta}{L}}$$

Pg-14.

Q1. $W_f = 2313\ lb = 1049.15\ kg$

$W_r = 1322\ lb = 599.64\ kg$

$L = 109$ inches $= 2.76\ m$

C.G = ? @ fore/aft position.

Conversion.

~~[illegible]~~ $1m = 39.37in$

~~1 lb = [illegible] kg~~

$1\ kg = 2.20\ lb$

So taking static position of vehicle

$$W_{rs} = \frac{W \cdot b}{L}$$

$$b = \frac{W_{rs} \cdot L}{W}$$

$$b = \frac{(599.64) \times 2.76}{(W_r + W_f)}$$

$$= \frac{(599.64) \times (2.76)}{(599.64 + 1049.15)}$$

$$\boxed{b = 1.003\ m}$$

* Traction and Tractive Effort

— The force available at the contact between drive wheel tyres and road is known as tractive effort.

— The ability of the drive wheels to transmit this effort without slipping is known as traction. Hence usuable tractive force never exceeds traction. The tractive effort relate to engine power as follows.

— Engine Torque (T_e)

$$\boxed{T_e = \frac{60 \times 1000 \times P}{2\pi N}}, \quad Nm$$

where P = B.P of Engine

T_e = Mean torque of Engine

T_W = Torque at drive wheels,

~~$T_W = (G R \times A) \eta_t T_e$~~

$$\boxed{T_W = G \cdot \eta_t \cdot T_e}$$

where, G = Overall Gear ratio of gear box.

η_t = Overall transmission efficiency.

Tractive effort,

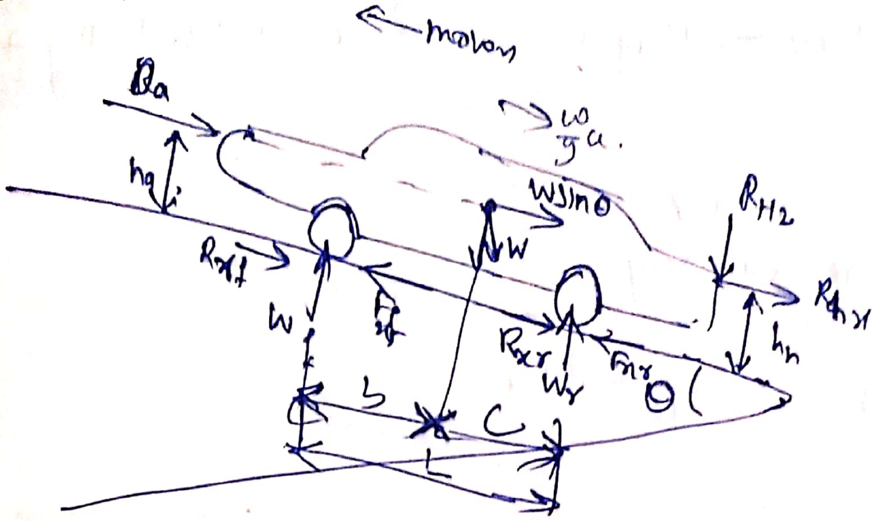
$$F = \frac{T_W}{r}$$

$$\boxed{F = \frac{T_e G . \eta_t}{r}} \quad N.$$

When tractive effort $F > R$, the total resistance on level road, the surplus tractive effort is utilized for acceleration, hill climbing (grade) and drawbar pull.

Equation of Motion and Maximum Tractive effort

15



aerodynamic Resistance D_A

rolling resistance of front tire $= R_{xf}$

— " — " — " rear — " — $= R_{xr}$

Drawbar load $= R_{hx}$

grade Resistance $\boxed{R_g = (W\sin\theta_s)}$

Tractive effort on front $= F_{xf}$

— " — " — Rear $= F_{xr}$

For rear wheel drive, $F_{xf} = 0$

— " — front — " — " , $F_{xr} = 0$

Equation of motion along longitudinal axis of vehicle

+ve ← → -ve

$$\therefore \boxed{\frac{W}{g}a = \cancel{R_{xf} + R_{xr} - F_{xf} - F_{xr} - R_{hx} - D_a + W}}$$

$$\therefore \frac{W}{g}a = F_{xf} + F_{xr} - R_{xf} - R_{xr} - R_{hx} - D_a - R_g$$

where, $\frac{W}{g}a = m \cdot \frac{d^2x}{dt^2}$. i.e linear acceleration of vehicle in longitudinal direction.

$$F_{xf} + F_{xr} - \left(R_{xf} + R_{xr} + R_{hx} + D_a + R_g + \frac{W a}{g}\right) = 0$$

or
we can say

$$\boxed{F = F_{xf} + F_{xr}}$$

where,

$$\boxed{F = \underbrace{R_{xf} + R_{xr}}_{R_x} + R_{hx} + D_a + R_g + \frac{W a}{g}}$$

This F is the total tractive effort
and R_x is the total Rolling Resistance of vehicle.

— To evaluate performance potential, the maximum tractive effort that the vehicle can develop has to be determined

two limiting factors to maxm tractive effort of road vehicle

- ① determined by Coeff. of Road adhesion & the normal load on axle.
- ② characteristics of power plant & transmission.

The smaller between this two determine performance potential of vehicle.

This ~~Coeff~~ front Factor can be determined by normal load. (i.e taking moment at A)

i.e,

$$W_f = \frac{1}{L}\Big[W_c \cos\theta - R_{hx} \cdot h_h - R_{hz} \cdot d_h - \frac{W}{g} a_x h - D_a h_a - W h \sin\theta\Big]$$

Similarly,

$$W_r = \frac{1}{L}\Big[R_{hx} \cdot h_h + W b \cos\theta + R_{hz}(d_h + L) + \left(\frac{W}{g} a_x \cdot h\right) + W h \sin\theta\Big]$$

let $h_h = h_a = h$ let $d_h = 0$ & neglect R_{hz}.

So,

$$\boxed{W_f = \frac{Wc}{L} - \frac{h}{L}\Big[R_{hx} + R_{hz} + \frac{W}{g} a_x + D_a + W \sin\theta\Big]}$$

$$\boxed{W_r = \frac{Wb}{L} + \frac{h}{L}\Big[R_{hx} + R_{hz} + \frac{W}{g} a_x + W \sin\theta\Big]}$$

Substituting the ~~rep~~ equation of tractive effort.

Eqs.

$$\boxed{W_f = \frac{Wc}{L} - \frac{h}{L}\big[F - R_x\big]}$$

$$\boxed{W_r = \frac{Wb}{L} + \frac{h}{L}(F - R_x)}$$

First term on R.H.S represent static load

2nd term represent dynamic load

The maximum traction effort that the tire-ground contact can support can be determined in term of road adhesion coefficient & vehicle parameters

For rear wheel drive

$$F_{max} = \mu W_r$$

$$= \mu \left[\frac{Wb}{L} + \frac{h}{L}(F_{max} - R_x) \right]$$

~~Fmax = ...~~

$$\cancel{= \frac{\mu Wb}{L} + \frac{\mu h F}{L} - \frac{\mu h R_x}{L}}$$

So,

$$F_{max} = \mu \left[\frac{Wb}{L} + \frac{h}{L}(F_{max} - R_x) \right]$$

$$F_{max} = \frac{\mu Wb}{L} + \frac{\mu h}{L} F_{max} - \frac{\mu h R_x}{L}$$

$$F_{max} - \left(\frac{\mu h}{L}\right) F_{max} = \frac{\mu Wb}{L} - \frac{\mu h R_x}{L}$$

$$F_{max}\left[1 - \left(\frac{\mu h}{L}\right)\right] = \frac{\mu W}{L}\left[b - \frac{R_x \cdot h}{W}\right]$$

$$\boxed{F_{max} = \frac{\frac{\mu W}{L}\left(b - \frac{R_x \cdot h}{W}\right)}{1 - \left(\frac{\mu h}{L}\right)}}$$

similarly for front wheel drive,

$$F_{max} = \mu W_f$$

$$= \mu\left[\frac{Wc}{L} - \frac{h}{L}(F_{max} - R_x)\right]$$

$$F_{max} = \mu\left[\frac{Wc}{L} - \frac{h}{L}(F_{max} - R_x)\right]$$

$$F_{max} = \frac{\mu Wc}{L} - \frac{\mu h F_{max}}{L} + \frac{\mu h R_x}{L}$$

$$F_{max} + \frac{\mu h F_{max}}{L} = \frac{\mu Wc}{L} + \frac{\mu h R_x}{L}$$

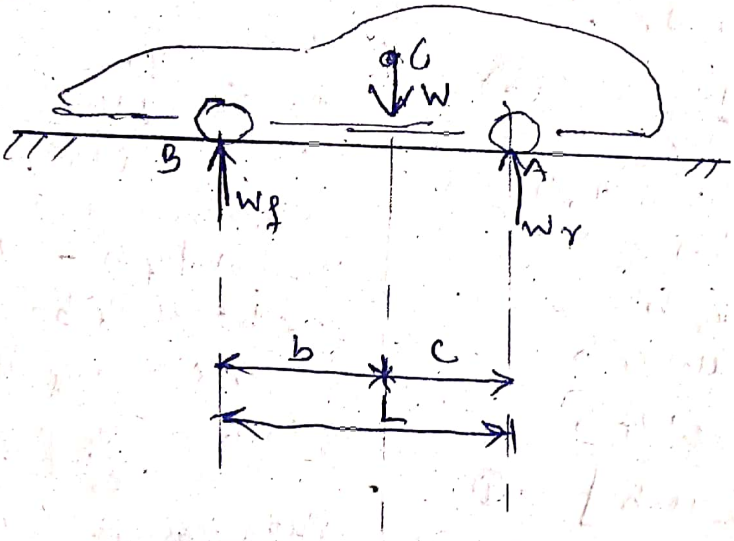
$$F_{max}\left[1 + \left(\frac{\mu h}{L}\right)\right] = \frac{\mu W}{L}\left[c + \frac{h R_x}{W}\right]$$

$$\boxed{F_{max} = \frac{\frac{\mu W}{L}\left(c + \frac{h R_x}{W}\right)}{1 + \left(\frac{\mu h}{L}\right)}}$$

– This is the condition for maximum Tractive effort

* **Distribution of weight :-** (4-wheel vehicle)

Forces acting on a 4-wheeled vehicle at rest are shown in fig. In this case only three independent equations can be formed to take care of four reactions at the wheels. Thus the problem is simplified by considering it as 2-wheeled vehicle i.e the reactions on both rear wheels are equal and also on both front wheels



vehicle is Stationary ie $v = 0$

$$\boxed{W = W_f + W_r} \quad \text{—} (1)$$

$$\Sigma M_B = 0 = -W_r \cdot L + W \cdot b$$

$$\boxed{W_r = \frac{W \cdot b}{L}} \quad \text{—} (2)$$

Putting equtn (2) in (1)

$$W = W_f + \frac{W \cdot b}{L}$$

$$W_f = W - \frac{W \cdot b}{L}$$

$$\boxed{W_f = W\left[1 - \frac{b}{L}\right]}$$

Ques A car weighing 21336.75N has a static weight distribution on the axles of 50 : 50. The wheel base is 3m and the height of C.G. above ground is 0.55m. If coefficient of friction on the highway is 0.6., Calculate the advantage of having rear wheel drive rather than front wheel drive as far as gradability is concerned, if engine power is not a limitation consider grade of 9°.

Ans Given :-

$W = 21336.75N$

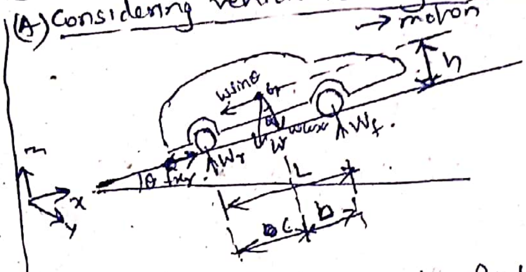
$b = c = \frac{3}{2} = 1.5m$

$L = 3m$

$h = 0.55m$

$\mu = 0.6$

(A) Considering vehicle having RWD :-



As vehicle is having RWD power will be at RW only hence friction will occur at RW only.

$f_{xr} = \mu \cdot W_r$

taking $\Sigma F_x = 0$.

$f_{xr} - W\sin\theta = 0$.

$\mu \cdot W_r - W\sin\theta = 0$

$\mu W_r = W\sin\theta$ — (1)

similarly,

$\Sigma F_z = 0$

$W\cos\theta = W_f + W_r$. — (2)

dividing equtn (2) by (1).

$$\frac{W\sin\theta}{W\cos\theta} = \frac{\mu \cdot W_r}{W_f + W_r}$$

$$\tan\theta = \frac{\mu \cdot W_r}{W_f + W_r} \quad \text{— (3)}$$

Taking moment at C.G, G

$W_f \cdot b - W_r \cdot c + f_{xr} \cdot h = 0$.

$W_r \cdot c = W_f \cdot b + f_{xr} \cdot h$

$W_r \cdot c = W_f \cdot b + \mu \cdot W_r \cdot h$

$W_f \cdot b = W_r \cdot c - \mu \cdot W_r \cdot h$

$$W_f = \frac{W_r}{b}\left[c - \mu \cdot h\right] \quad \text{— (4)}$$

now putting values in equtn (4)

$$W_f = W_r\left[\frac{1.5}{1.5} - \frac{(0.6)(0.55)}{1.5}\right]$$

$W_f = W_r(0.78)$ — (5).

Similarly putting values in equtn (3) from equtn (5)

$$\tan\theta = \frac{(0.6)W_r}{(0.78)W_r + W_r}$$

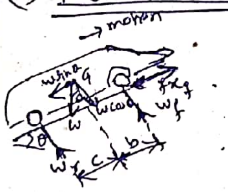
$$\tan\theta = \frac{0.6}{1.78}$$

$\tan\theta = 0.337$.

Percentage grade $= 0.337 \times 100$

$= 33.7\%$.

(B) Now Considering FWD



$W_f + W_r = W\cos\theta$ — (1)

$f_{xf} = \mu W_f$.

$f_{xf} - W\sin\theta = 0$

$f_{xf} = W\sin\theta$

$\mu W_f = W\sin\theta$ — (2)

Similarly,

$$\tan\theta = \frac{\mu W_f}{W_f + W_r} \quad \text{— (3)}$$

Taking moment at C.G.

$W_f \cdot b + f_{xf} \cdot h - W_r \cdot c = 0$

$W_f \cdot b + \mu \cdot W_f \cdot h - W_r \cdot c = 0$

$W_r \cdot c = W_f \cdot b + \mu \cdot W_f \cdot h$

$$W_r = \frac{W_f}{c}\left[b + \mu \cdot h\right] \quad \text{— (4)}$$

Putting values

$$W_r = W_f\left[\frac{1.5}{1.5} + 0.6(0.55)\right]$$

$W_r = 1.22 W_f$ — (5)

Putting equtn (5) in equtn (3)

$$\tan\theta = \frac{(0.6)W_f}{W_f + 1.22W_f}$$

$\tan\theta = 0.27$ or 27%.

Therefore the car having RWD can negotiate the grade

$$= \frac{33.7 - 27}{27} = 24.8\%$$

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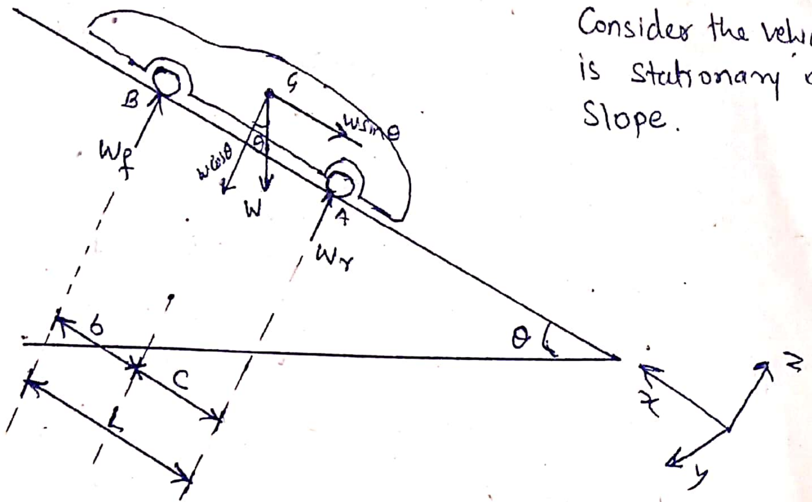
* Stabality of vehicle on slope:-



Consider the vehicle is stationary on slope.

By taking moment @ A,

$$W_f = \frac{1}{L}\left[W.c.\cos\theta - R_{hx}.h_h - R_{hz}.d_h - \frac{W}{g}a_x.h - D_a h_a - Wh\sin\theta\right]$$

But as vehicle is stationary,

$D_a = 0$ — Aerodynamic Resistance

$\frac{W}{g}a_x = 0$ — Inertia force

$R_h = 0$ — drawbar Pull.

Hence,

$$W_f = \frac{W}{L}\left[c.\cos\theta - h.\sin\theta\right] \quad \text{— (1)}$$

Similarly, Taking moment @ B.

$$W_r = \frac{1}{L}\left[R_{hx}.h_h + W.b.\cos\theta + R_{hz}.(d_h+L) + \left(\frac{W}{g}a_x.h\right) + (D_a.h_a) + W.h\sin\theta\right]$$

Considering vehicle stationary.

$$W_r = \frac{W}{L}\left[b\cos\theta + h\sin\theta\right] \quad \text{— (2)}$$

equation (1) and (2) are normal loads on front and rear axle, when vehicle is stationary.

Similarly, If we increase the slope angle (θ) gradually, then a situation arises when,

a) Either the vehicle is about to overturn, or.

b) The vehicle is about to slide down the slope.

Considering the case (a) i.e vehicle overturn.

Case (a) :-

If vehicle overturns than the point of contact at B will be lost, and hence the

reaction of front axle will be zero,
hence,

$W_f = 0$. — for limiting condition of overturn.

so we can write the equation ① as,

$$0 = \frac{W}{L}[c \cdot \cos\theta_L - h \sin\theta_L]$$

as, $\frac{W}{L} \neq 0$

$$c \cdot \cos\theta_L - h\sin\theta_L = 0$$

$$c \cdot \cos\theta_L = h \sin\theta_L$$

$$\boxed{\tan\theta_L = \frac{c}{h}}$$

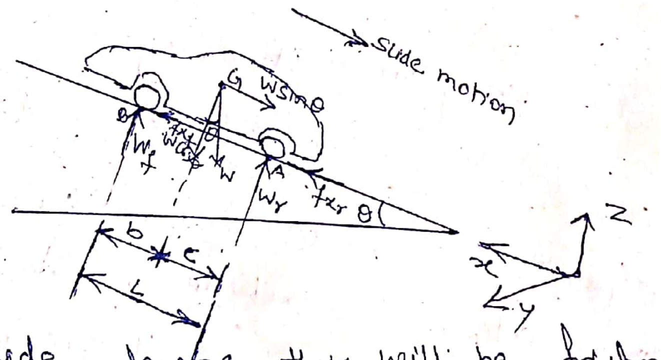
where θ_L is the limiting angle of slope for overturning.

This indicates that at point of overturning, the line of action of weight W passes through the point of Contact B.

Case(B) :- The vehicle is about to slide down the Slope:-

Considering,

~~$\Sigma F_x = 0$~~



As vehicle slide downs, there will be friction ~~opposi~~ force f_{xf}, f_{xr} in opposite direction.

$$\Sigma F_x = -W\sin\theta + f_{xf} + f_{xr} = 0$$

$$f_{xf} + f_{xr} = W\sin\theta \quad — ③$$

$$\Sigma F_y = 0 = W_f + W_r - W\cos\theta = 0$$

$$W_f + W_r = W\cos\theta \quad — ④$$

Scanned with CamScanner

The limiting value of θ is given by,

$$W \cdot \sin\theta = f_{xf} + f_{xr}$$

The value of $(f_{xf} + f_{xr})$ can be determined under the following conditions. Let the brakes be applied to prevent this situation.

Then two cases may arise.

Case I :- The brakes are not efficient enough to prevent the wheels from turning before they slide. In this case the limiting value of θ can be determined by the brake torques available.

If, T_f and T_r are the braking torques at front and rear wheels respectively, then,

$$\left.\begin{aligned} T_f &= f_{xf} \cdot r \\ \text{and} \quad T_r &= f_{xr} \cdot r \end{aligned}\right] \text{ where } r \text{ is the radius of wheel.}$$

Total Braking torque,

$$T_f + T_r = (f_{xf} + f_{xr}) \cdot r \quad \text{—— (5)}$$

~~Tf+Tr~~ from equation (3) and substituting equation (5) in (5)

$$T_f + T_r = (W \sin\theta_L) r$$

$$\boxed{\sin\theta_L = \frac{(T_f + T_r)}{W \cdot r}}$$

Case II The brakes are sufficiently powerful for the coefficient of adhesion, μ to limit the sliding of the vehicle. then,

$$f_{xf} = \mu W_f$$

$$\text{and } f_{xr} = \mu W_r$$

total,

$$f_{xf} + f_{xr} = \mu (W_f + W_r) \quad \text{—— (6)}$$

from equation (4), substituting in equation (6)

$$f_{xf} + f_{xr} = \mu (W \cdot \cos\theta)$$

Similarly from equtn. (3), substituting in equtn (6)

$W \cdot \sin\theta_L = \mu \cdot W \cdot \cos\theta_L$

$$\boxed{\tan\theta_L = \mu}$$

It should be noted that when the vehicle is being driven up, the angle of overturning is, in general, smaller than in the present case and also the condition of instability becomes different from discussed above.

Ques. A vehicle of total weight 49050 N is held at rest on a slope of 10°. It has a wheel base of 2.25 m and its centre of gravity is 1.0 m in front of rear axle and 1.5 m above ground level.

Find
a) What are normal reactions at the wheels?
b) Assuming that sliding doesn't occur first, what will be the angle of slope, so that the vehicle will overturn?
c) Assuming all wheels are to be braked, what will be the angle of slope so that the vehicle will begin to slide if coeff. of adhesion between tire and ground is 0.35?

Sol: Given:

$W = 49050\ N$

Slope $= 10^\circ$

$L = 2.25 m$

$C = 1 m$

$h = 1.5 m$

$W_f + W_r = W \cos\theta$

$W_f + W_r = 48304$ — (1)

$W_f = \frac{W}{L}[C \cdot \cos\theta - h \sin\theta]$

$$\boxed{W_f = 27145\ N}$$

$$\boxed{W_r = 21159\,N}$$

(b) Slide not occurs, when will vehicle overturns.

$$\tan\theta_L = \frac{c}{h}$$

$$\boxed{\theta_L = 39^\circ 49'}$$

(c) when Brakes are applied

$$\tan\theta_L = \mu$$

$$\tan\theta_L = 0.35$$

$$\boxed{\theta_L = 19^\circ 17'}$$

Equivalent ungeared System

compliance → according.

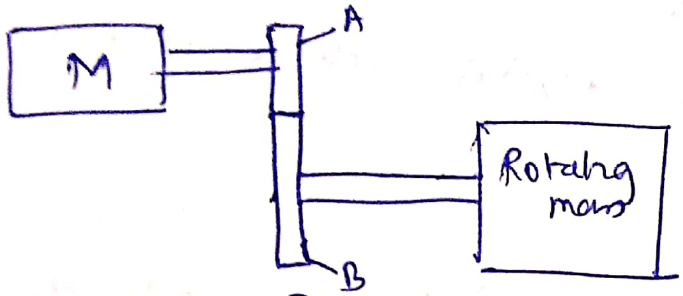


Fig - ① - geared system

~~Gear ratio or~~ Speed ratio

$$= \frac{\omega_{in}}{\omega_{o/p}} = \frac{\alpha_{in}}{\alpha_{o/p}}$$

So,

gear Speed ratio $= \frac{\omega_A}{\omega_B} = \frac{\alpha_A}{\alpha_B} = \frac{Z_B}{Z_A} \quad \frac{D_B}{D_A} = \frac{r_B}{r_A}$

gear Torque ratio. $= \frac{T_B}{T_A} = \frac{O/P}{I/P}$.

So

let, gear ratio $= G$

Consequently there is no need to take distinction in notation between gear speed ratio and gear torque ratio one can just say gear ratio which one can require to have o/p ~~or~~ speed or O/P torque.

~~$G = \frac{\omega_B}{\omega_A}$~~

and

Gear torque ratio, $G = \frac{T_B}{T_A}$

So,

$$\omega_B = \frac{1}{G}\omega_A$$

$$T_B = G\, T_A$$

Now in order to find an ungeared System equivalent to geared system of fig ①.

So Torque on gear B,

$$T_B = I_B \alpha_B$$

and we known that

$$T_A = I_A \alpha_A$$

~~So we can say that~~

$$\cancel{T_A = I_A \cdot \left(\frac{1}{G}\alpha_B\right)}$$

Now let the rotational inertia I_A of the motivating source i.e gear A is ~~taken~~ ~~assumed~~ ignored ~~zero~~. then, Calculating backwords from the above O/P Torque about B the necessary input torque about A will be.

$$T_A = I_B \cdot \left(\frac{\alpha_B}{G}\right)$$

$\because G = \frac{T_B}{T_A}$

$T_A = \frac{T_B}{G}$

$$\boxed{T_A = \frac{I_B \cdot \alpha_B}{G}}$$

But we know that I_A will not be zero.

So ~~total~~ torque at A will be

$$T_A = I_A \cdot \alpha_A + I_B \cdot \left(\frac{\alpha_B}{G}\right)$$

but we know that $\alpha_B = \frac{\alpha_A}{G}$

$G = \frac{\alpha_A}{\alpha_B}$

so, $\alpha_B = \frac{\alpha_A}{G}$

So, $$T_A = I_A \cdot \alpha_A + \frac{I_B \cdot \alpha_A}{G^2}$$

$$\boxed{T_A = \left(I_A + \frac{1}{G^2} I_B\right)\alpha_A}$$

Thus the engeared system equivalent to the ~~gear~~ gear system has an effective ~~rotat~~ rotational inertia about A of $\left(I_A + \frac{1}{G^2} I_B\right)$.

If gear set efficiency is to be included then,

$$\boxed{T_A = \left[I_A + \frac{I_B}{G^2} \cdot \frac{1}{\eta}\right]\alpha_A}$$

$\eta \Rightarrow$ represents the loss in energies

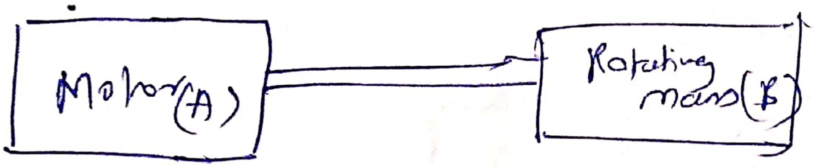


Fig-① Equivalent Ungeared system w.r.t A

If anyone want to find equivalent inertia of the geared system about B then the process will be

① Take B as driver, the torque about B to drive A & B will be,

$$T_B = (I_B + G^2 I_A)\alpha_B$$

If gear efficiency is to be included

$$T_B = \left[I_B + \frac{G^2 I_A}{\eta}\right]\alpha_B$$

This torque T and effective ~~rot~~ rotational inertia Ie of the above discussion of rotational behaviour ($T = I_e \alpha$) have counterparts to be found ~~in~~ translational relationship $F = m_e a$

For automobile,

In acceleration the torque about the drive wheel axle reduces to a traction force at the tire/ground contact patches. hence it is given by,

$$\boxed{F = \left\{\frac{T_e\cdot(\eta_t\cdot N_T)(N_f\cdot\eta_f)}{r}\right\} - R_x - D_A}$$

or we can write,

$$\boxed{F = \left[\frac{T_e \cdot N_{tf} \cdot \eta_{tf}}{r}\right] - R_x - D_A}$$

where

N_{tf} = Combined gear ratio of Transmission drive & final drive

η_{tf} = Combined efficiency of transmission drive & final drive.

T_e = engine torque.

R_x = Rolling resistance

D_A = Aerodynamic resistance.

An automobile is a complex assembly of variegated components. Some of which are accelerated translationally and rotationally.

To account for overall effect of this ~~Vartge~~ variegated behaviour the concept of effective mass me must be substituted for the fundamental mass m

To do so gearing relationship are required

$$m_e = \frac{W}{g} + \frac{I_a + I_e (N_t \cdot N_f)^2 + I_t (N_f)^2 + I_f}{R_x^2 \cdot g}.$$